DETERMINATION OF THE DYNAMIC STRESS INTENSITY FACTOR USING ADVANCED ENERGY RELEASE EVALUATION

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1. INTRODUCTION

The determination of the stress intensity factor (SIF) appears to be one of the main subjects of the fracture mechanics. A significant progress in the methods of the linear fracture mechanics (LFM) has been made during the past two decades. A lot of numerical and analytical techniques and methods for the SIF estimation with an satisfactory accuracy and different computational efforts assuming static or monotonically increased external loads are known. Recently the SIF evaluation in the case of dynamic external loading is studied intensively. In the LFM the energy release rate *G* can be used to calculate the stress intensity factor *K* [1], [2]. The exact values of *K* are known only for a limited number of cases assuming an idealized shape or special boundary conditions. For more complex structures and types of loading the SIF can be studied numerically using different techniques such as the finite element method (FEM) or the boundary element method (BEM). For specific problems in the LFM the BEM provides the best accuracy and the lowest number of unknown parameters that are to be determined. However for a wide variety of problems including a material nonlinearity and dynamic external loads, the FEM appears to be an more general and versatile computational approach. Some new developments and upgrades of this method enable the researcher to calculate fracture mechanics parameters through the usual procedures of the FEM. For example the known *J*-integral and the released energy per unit released length can be evaluated numerically [1], [2]. At present the nonlinear fracture mechanics (NFM) is developing too.

In this study a simple effective procedure practically based upon the FEM for determination of the dynamic stress intensity factor (DSIF) depending on the input frequency and using an advanced strain energy release evaluation by the simultaneous release of a set of fictitious nodal spring links near the crack tip is developed and applied. The DSIF is expressed in terms of the released energy per unit crack length similarly to the case of a static loading. The formulations of the LFM are accepted. This approach is theoretically based upon the eigenvalue problem for assessment of the spring stiffnesses and on the modal decomposition of the crack shape. The inertial effects are included into the released energy. A linear elastic, homogeneous and isotropic material, time-dependent external loading of harmonic type and steady state undamped response of the structure are assumed. The procedure allows the opening (first), sliding and mixed modes of the structure fracture to be studied in demand of K. The shear mode is neglected. The decreased dynamic stiffness is taken into account. This technique requires a fine mesh near the crack tip.

2. THEORY AND AN ALGORITHM

According to Ref. [1] the released energy G per unit crack length is defined by the following expression (Fig. 1):

$$G = \lim_{\Delta a \to 0} \frac{1}{\Delta a} \int_0^{\Delta a} \int_0^{u^0(x)} t(u)\,du\,dx, \tag{1}$$

where *Δa* is the measure of the crack length growth and *t(u)* are the tractions, acting on the surface that is to be released. These tractions are assumed to be dependent on the crack opening displacements *u* and u^0 denotes the displacements of the traction-free surface. Note that the crack length increment *Δa* tends to zero in the expression (1). From the numerical point of view *Δa* will have only a finite value and it will never reach zero. The relationship between the normal stresses σ and the traction is

$$t = \sigma\delta_x \qquad t \rightarrow [\mathrm{kN/m}], \qquad G \rightarrow [\mathrm{kNm/m}], \tag{2}$$

where δ

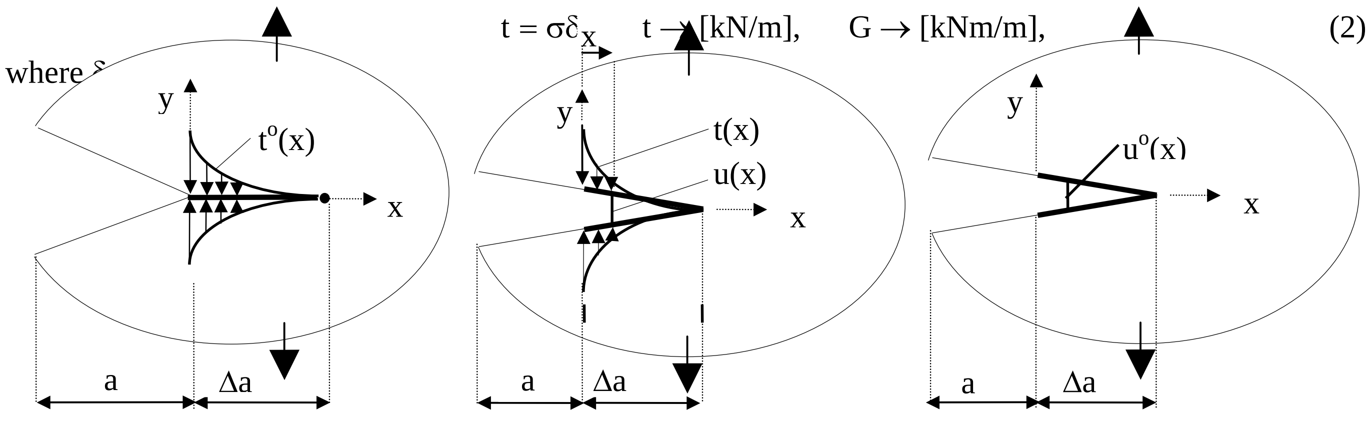


Fig. 1. Initial, Intermediate and Final States Formulation

Following the above definition for the released energy per unit length one may express *G* in a matrix form (Fig. 2) when more than one discretized link elements are located on a length Δa, namely

$$G = \int_{\mathrm{state}\,0}^{\mathrm{state}\,1} \{t\}^T \{du\}, \qquad [\mathrm{kNm/m}] \tag{3}$$

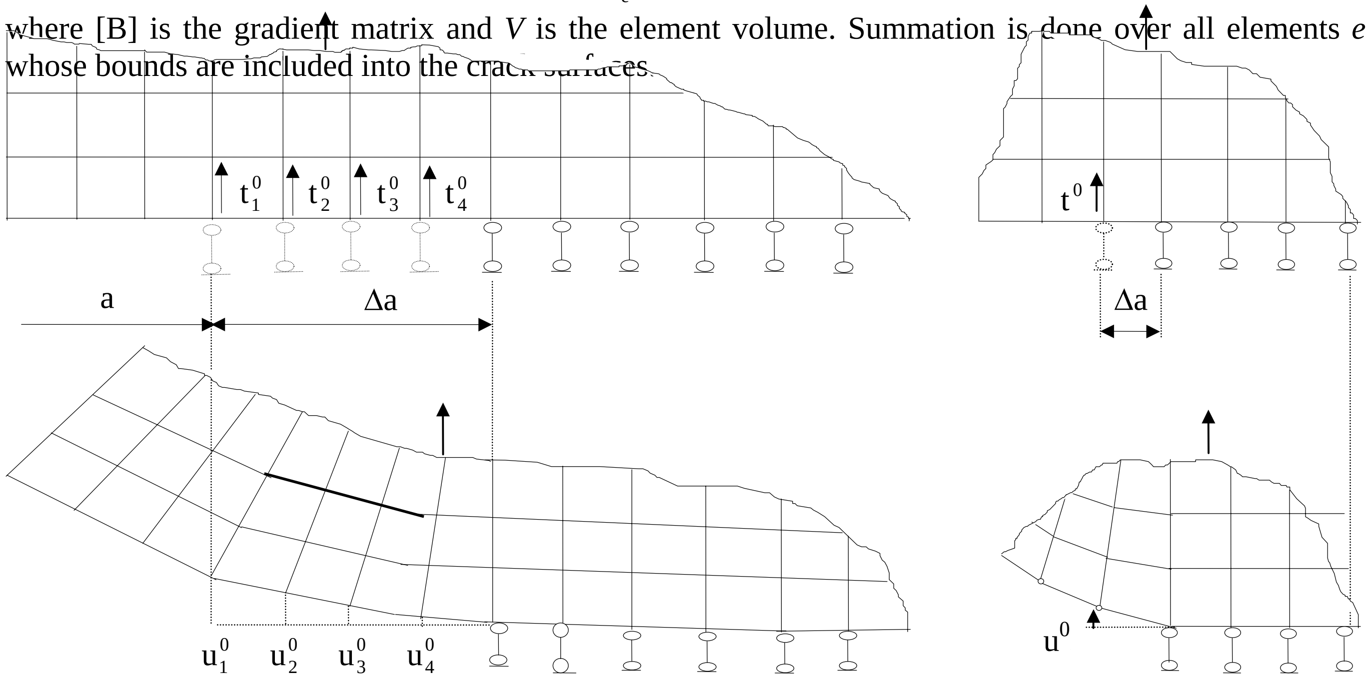
Here {t} and {du} denote the traction vector and the opening displacements vector respectively. An similar expression is used in [1] but for a single link in the crack. It is suggested that if several links are considered, the matrix formulation is required. Note that the integral limits are related to the initial *state 0* and with the final *state 1*. It can be proven that in the LFM the value of the integral in Eq.(3) is dependent only on these both states and not on the loading path. In the NFM the value of *G* is path dependent. The relation between the tractions and internal forces {f} associated with the released crack surfaces is as

$$\{t\} = \frac{1}{\delta}\{f\}. \tag{4}$$

According to the finite element approach we have

$$\{f\} = \sum_e \int [B]^T \{\sigma\} dV, \tag{5}$$

where [B] is the gradient matrix and *V* is the element volume. Summation is done over all elements *e* whose bounds are included into the crack surfaces.



a) Simultaneous Release of a Group of Links b) Release of a Single Link

Fig. 2. Simultaneous Release of Several Links and Release of a Single Link

Figure 3 illustrates the process of simultaneous release of a set of disctetized links attempting to simulate the opening of the crack. Considering only a single node, we may imagine that two forces are applied there. The first force will appear in a vertically restrained node. The second force takes into account the effect of decreasing when the link is moved along the first force. This force is called *fictitious force* and it is indicated as a force, acting in a spring with a negative spring coefficient. For all links one may write the following matrix equilibrium equation:

$$\left([s(\theta)] - [c(\theta)]\right)\{u\} = \{0\}\ , \tag{6}$$

where [c] is a diagonal matrix, containing all spring stiffnesses and [s] is the stiffness matrix of the released length. It is expediently the matrix [s] to be determined using the flexibility matrix [d] as follows

$$[s(\theta)] = [d(\theta)]^{-1}\ .$$

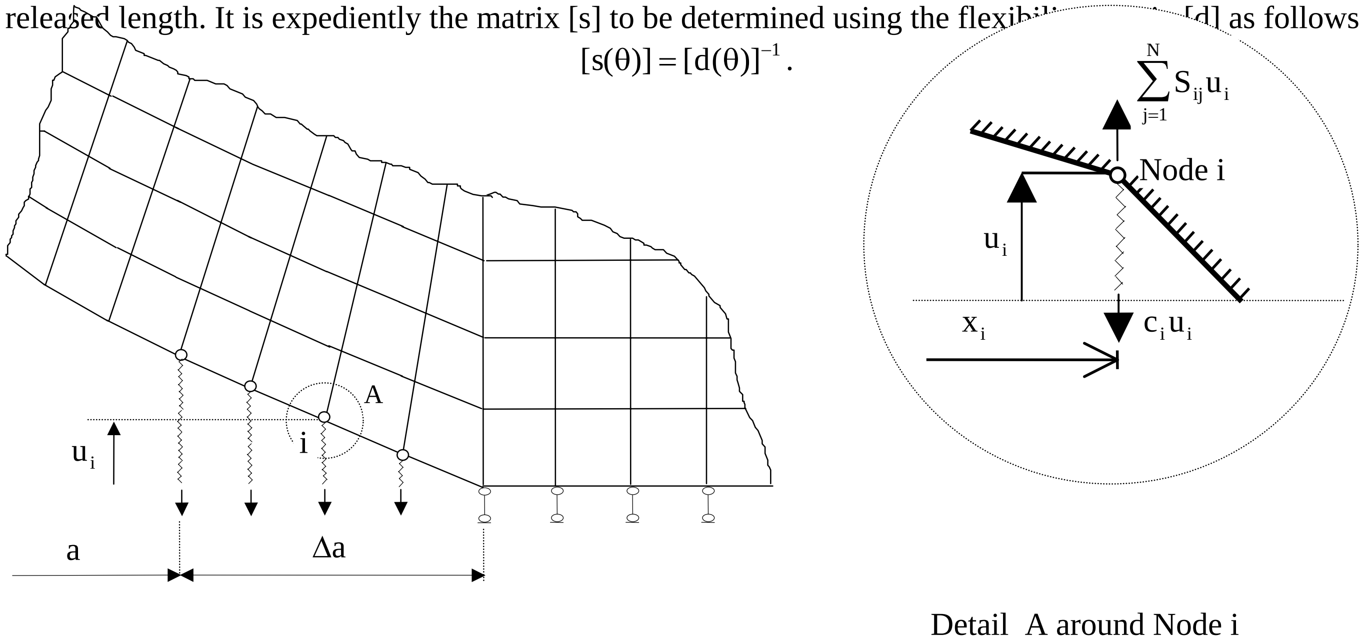


Fig. 3. Implementation of Fictitious Springs in Places Where Tractions Are To Be Released

Note that all these matrices are dependent on the input frequency θ. Obviously Eq. (6) is related to the eigenvalue problem. The quantities that should be determined are the fictitious spring stiffnesses (eigenvalues) and the corresponding mode shapes (eigenvectors) (Fig. 4).

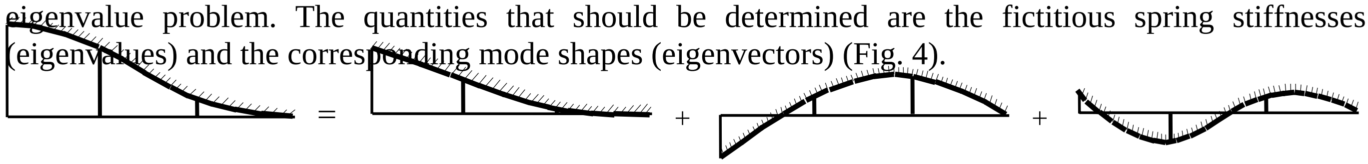


Fig. 4. Representation of Crack Opening Displacements As a Sum of Modal Terms

Following the LFM formulation and considering Fig. 5a, the following relationship is hold :

$$\{t\} = \left\{t^0\right\} - [c]\{u\}\ . \tag{7}$$

In the light of NFM and using Fig. 5b, Eq.(6) is written in the following incremental form:

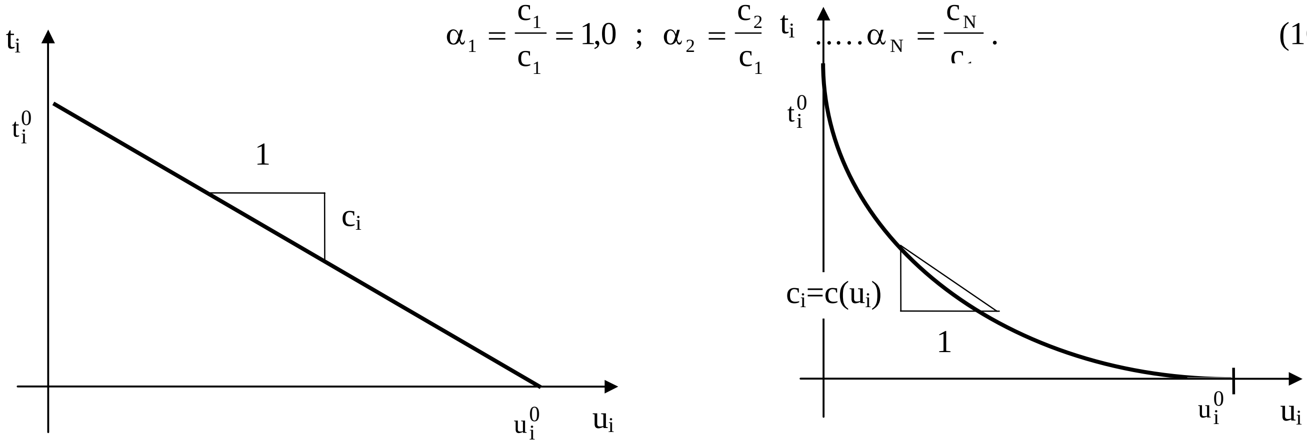
$$\left([s] - [c]\right)\{du\} = \{0\}. \tag{8}$$

Here the spring stiffness matrix [c] is represented by

$$[c] = \begin{bmatrix} c_1 & & & \\ & c_2 & & \\ & & \ddots & \\ & & & c_N \end{bmatrix} = c\begin{bmatrix} \alpha_1 & & & \\ & \alpha_2 & & \\ & & \ddots & \\ & & & \alpha_N \end{bmatrix} \tag{9}$$

with the following notations

$$\alpha_1 = \frac{c_1}{c_1} = 1{,}0 \;\; ; \;\; \alpha_2 = \frac{c_2}{c_1} \; \ldots . \alpha_N = \frac{c_N}{c_1} \, . \tag{10}$$



a) Traction-Displacement Diagram in LFM b) Traction-Displacement Curve in NFM

Fig.5. The Relationship Between Traction and Crack Opening Displacement Used in LFM and in NFM

In this study the simplest form of the matrix [c] is accepted. It has the form

$$[c] = c[I] \, , \tag{11}$$

where [I] denotes the identity matrix. Replacing Eq.(7) into Eq.(3) the following result in LFM is derived:

$$G = \int_{\text{state 0}}^{\text{state 1}} \{t\}^T \{du\} = \{t^0\}^T \{u^0\} - \frac{1}{2} \{u^0\}^T [c] \{u^0\} \, . \tag{12}$$

Taking into account that

$$\{t^0\} = [c] \{u^0\} \, , \tag{13}$$

the released energy becomes

$$G = \frac{1}{2} \{u^0\}^T [c] \{u^0\} \, . \tag{14}$$

In the NFM the released energy can be obtained in an incremental form, based on current update of the cumulated energy by the corresponding increment, namely

$$G_{new} = G_{old} + \{t\}^T \{\Delta u\} = G_{old} + \Delta G \, . \tag{15}$$

In the LFM the SIF for the first (opening) mode K_I is related by the released energy by the expression

$$K_I = \sqrt{\frac{GE}{\left(1-\nu^2\right)}} \, , \tag{16}$$

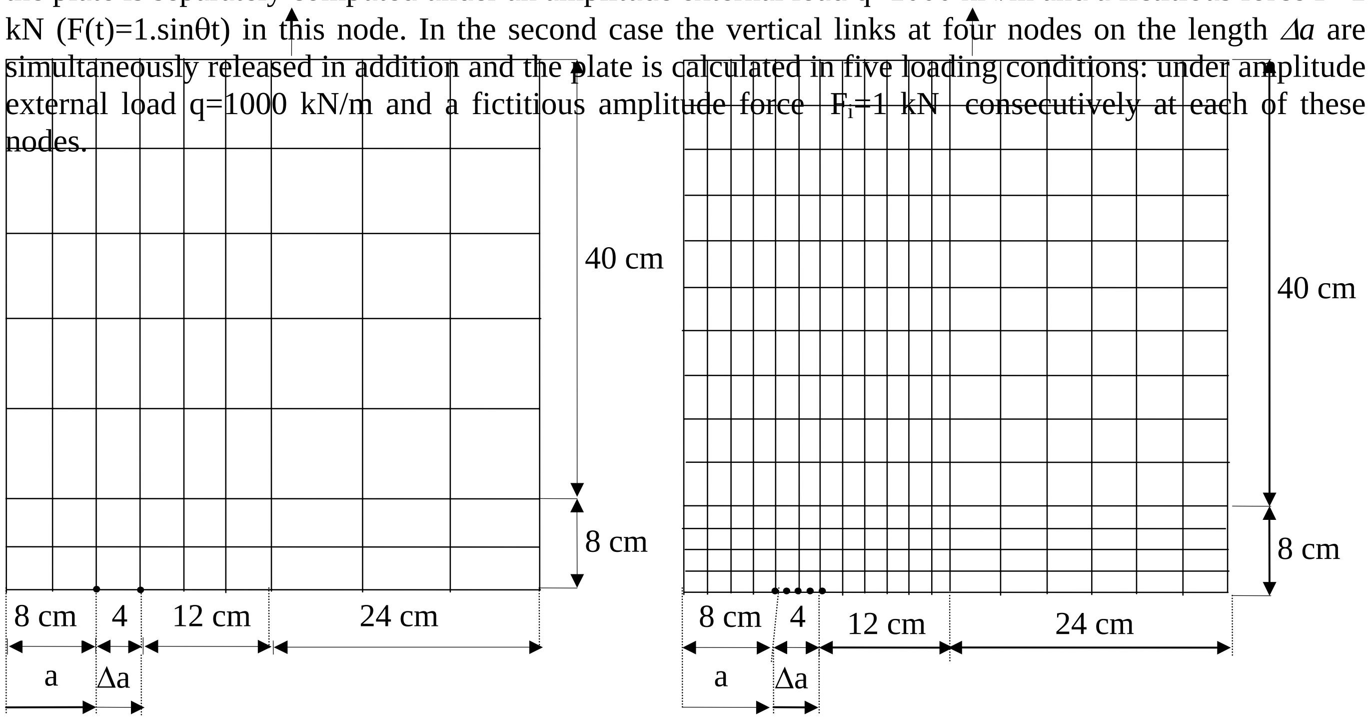
where E is the Young's modulus and ν is the Poisson's ratio. Note that inertial effects are accounted for by the released energy in the above result.

3. NUMERICAL EXAMPLE, RESULTS AND DISCUSSION

The application of the proposed procedure is examined and demonstrated by an numerical test example of a simple square unsupported notched steel plate 0,96 m in side and with a central horizontal crack. The plate is subjected to self-equilizing uniform in-plane, sine in time, tension loads of an intensity q(t)=qsinθt, applied on the two opposite sides. Using double symmetry a quarter of the plate (Fig. 6) in the plane stress state within the middle horizontal plane is calculated considering the undamped steady state response. The opening mode of fracture is studied only. The DSIF is computed using a coarse mesh and a single node release for the released energy computation as well a fine mesh and simultaneous release of four links to obtain more accurate values. The released length *Δa* in both cases is the same. The following material properties, geometrical and loading parameters are specified:

$E=2{,}1.10^8$ kN/m^2, $\nu=0{,}3$, $\delta=0{,}01$ m, $q(t)=q\sin\theta t=1000\sin\theta t$ kN/m, $q=1000$ kN/m, $\sigma(t)=\sigma\sin\theta t=100000\sin\theta t$ kN/m^2, $\sigma=100000$ kN/m^2, $a=0{,}08$ m, $\Delta a=0{,}04$ m.

The natural frequences of the structure are computed by the Ritz's vector analysis and by the eigenvalue analysis (through the Jacobi's method) using the SAP2000 computer program. The quadrilateral ASOLID finite element (FE) based upon an isoparametric formulation is used for modeling. It has got of 4 nodes (8 DOF) and 9 nodes (18 DOF) in the two cases respectively. The meshes includes 80 and 1073 nodes. Four FE with different dimensions are used. The nodal links on the whole crack length $a+\Delta a=0{,}12$ m are released. The links on the length a are initially released but the links on the length Δa are released in addition. In the first case the vertical link at the initial node of the length Δa is additionally released and the plate is separately computed under an amplitude external load q=1000 kN/m and a fictitious force F=1 kN ($F(t)=1.\sin\theta t$) in this node. In the second case the vertical links at four nodes on the length Δa are simultaneously released in addition and the plate is calculated in five loading conditions: under amplitude external load q=1000 kN/m and a fictitious amplitude force $F_i=1$ kN consecutively at each of these nodes.



a) A Coarse Mesh Used b) A Refined Mesh Used

Fig. 6. Numerical Example - A quarter of A Notched Steel Cracked Plate Studied and Two Meshes Used

The following values of the natural frequences $\omega_i^{(1)}$ and $\omega_i^{(4)}$ are obtained if a single or four links are released respectively:

$\omega_1^{(1)}\approx 2150$ Hz, $\omega_2^{(1)}\approx 2610$ Hz, $\omega_3^{(1)}\approx 2950$ Hz, $\omega_4^{(1)}\approx 4413$ Hz, $\omega_5^{(1)}\approx 5036$ Hz, $\omega_6^{(1)}\approx 6166$ Hz;

$\omega_1^{(4)}\approx 2126$ Hz, $\omega_2^{(4)}\approx 2640$ Hz, $\omega_3^{(4)}\approx 2941$ Hz, $\omega_4^{(4)}\approx 4439$ Hz, $\omega_5^{(4)}\approx 5225$ Hz, $\omega_6^{(4)}\approx 6286$ Hz.

Ranging the load frequency θ in the steady state analysis in limits from 0,0 up to $1{,}1\omega_6^{(4)}$ the crack opening vertical displacement at the released 1 or 4 nodes on the length Δa in each unique state and also under an external uniform opening load of an amplitude intensity q are determinate. Within the resonance zones the densely values of θ are considered. The eigenvalue problem is solved by an author's program applying the Jacobi's method and then the fictitious nodal spring reactions (stiffness coefficients) and the crack modal vertical displacements at the released nodes for every examined value of θ are calculated. The released energy per unit crack length G and the DSIF are obtained by the same program for the same values of θ.

The results are analyzed. Comparisons with the known exact results from a static loading are presented in Fig. 7. The first three natural frequences $\omega_i^{(4)}$ and $\omega_i^{(1)}$ (i=1, 2, 3) are only used to construct these two curves in Fig. 7 where θ is ranging between 0,0 and $1{,}2\omega_3^{(4)}$. The ratio K_I *-dynamic/*K_I *-exact* in a logarithmic scale is represented versus the input frequency/fundamental frequency ratio (θ/ω_1). K_I-*dynamic* is calculated by the Eq.(16) but K_I-*exact* $=\sigma\sqrt{\pi a}$ is taken as a static exact value of the SIF for an infinite notched plate in the two orthogonal directions, known in [2].

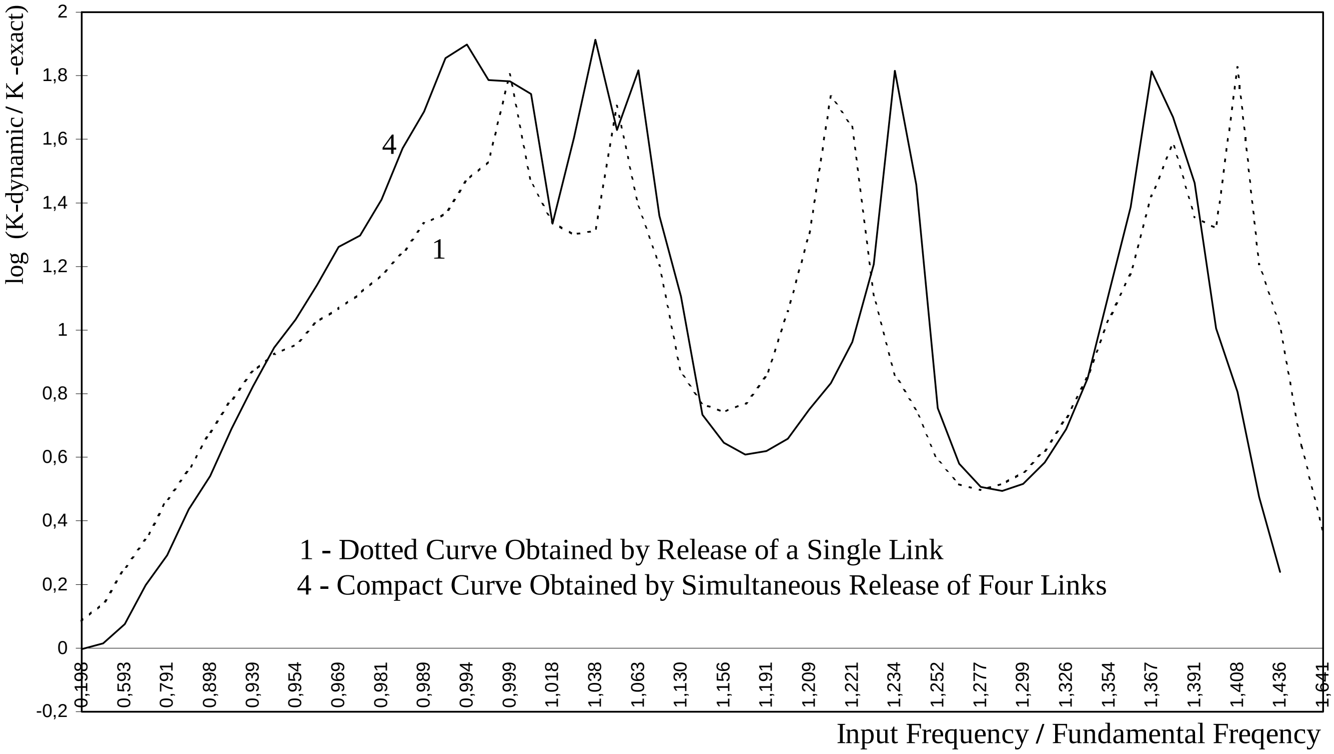


Fig. 7. Dynamic Stress Intensity Factor Ratio

The DSIF is strongly dependent on the input frequency. It is seen from the Fig. 7, that when the input frequency θ is equal to zero (static analysis), the simultaneous release of four links provides the value of the SIF that is very close to the exact one (an relative error of 1,0 % only). Alternatively, the procedure with release of a single link produces an error of about 16,6 % in the same result. The mesh refinement near the crack tip as well the simultaneous release of larger number of nodes along the crack provides more accurate values of the DSIF. These values are significantly larger than the values of the static stress intensity factor obtained by the same meshes but under the appropriate static load of an intensity equal to the amplitude dynamic load value. It is clear from the Fig. 7 also, that the dynamic loads may lead to an considerable increase of the SIF and it should be taken into account in the design. Significant peaks of the DSIF are observed near the natural frequencies. The peak values of the two curves are slightly removed in the time domain because of the values of the corresponding natural frequences and also the rezonance zones in these two cases are different.

4. CONCLUSIONS

The proposed general procedure is practicable and reliable enough and its application can be extended to more complex cases. The presented FEM technique is an effective tool for the DSIF estimation in the LFM. This rational and powerful technique requires a mesh refinement in the vicinity of the crack tip. The procedure extends the FEM application in the fracture mechanics. This approach can be more effective in the NFM especially in analysis and design of mechanical engineering structures such as transport machines, motor industry and others. Direct time integration operators in the frequency domain are then applied. In the NFM the BEM is not powerful and versatile and even can be inapplicable for the

DSIF determination. The essential contribution in this paper is the development and application of a FEM technique for the DSIF calculation using the released energy by simultaneous release of a few nodes near the crack tip.

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